

# ANALYSIS OF VERTICAL MAGNETIC SUSPENSIONS

W. STEPHEN CHEUNG , CARL H. LEYH, AND ROGERS C. RITTER

DEPARTMENT OF PHYSICS, UNIVERSITY OF VIRGINIA  
CHARLOTTESVILLE, VIRGINIA 22901 U.S.A.

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## ANALYSIS OF VERTICAL MAGNETIC SUSPENSIONS

W. Stephen Cheung<sup>†</sup>, Carl H. Leyh, and Rogers C. Ritter  
Department of Physics, University of Virginia  
Charlottesville, VA 22901

The vertical magnetic suspension, as used for a low friction bearing in the Fundamental Measurements Laboratory at the University of Virginia, is analyzed as a simplified physical system. The separation of forces into equilibrium and control forces, in conjunction with properties of the magnetic field, leads to simple equilibrium equations. One practical case is evaluated as an example. The control properties of the servo loop are first subjugated into a model in which the overall behavior is that of damped harmonic motion of the suspended object. A special, parametric suspension with improved properties is discussed briefly. Finally, the servo loop elements, in simplified form, are related to parameters of the model differential equation.

## 1. INTRODUCTION

Magnetic suspension is a physical process through which a given object (often ferromagnetic) is levitated without solid body contact or attachment to anything else. A teaching model was described by Beams,<sup>1</sup> but this gave very little quantitative information of the physics of the magnetic suspension.

Basically, one can understand a magnetic suspension from Fig. (1). Here, the position transducer (an incandescent light source and a photodetector in Beam's paper) picks up any movement of the ball and causes the feedback circuitry to send an appropriate amount of current to the solenoid to retain the ball's original position. Beams was able to control the short-term vertical motion of the sphere to less than a wavelength of visible light.<sup>2</sup> Several position transducers have been adopted in magnetic suspensions; they include optical sensing (discussed further below), Q coil sensing<sup>3</sup>, capacitance sensing<sup>4</sup>, and resonant frequency modulation\*. It must be pointed out that each of these schemes has its vulnerabilities, limitations, sensitivities, and applicabilities. For example, the usual optical method is vulnerable to ambient light change, temperature drift, electromagnetic disturbances, etc. The signal processing after the transducer is independent of the detection scheme while the limitation and vulnerability are not.

The magnetic suspension has been used as an ultra-low friction bearing for many years<sup>5,6</sup>. In recent times it has found use in heavy machinery in a number of complicated forms.<sup>7</sup> Our laboratory has been employing magnetic suspensions in various gravitational experiments<sup>8,9,10</sup> as well as in biophysical measurements of viscosity and density of virus solutions.<sup>11</sup> Figure 2 is a picture of one of these suspensions.

\*This scheme has just been successfully demonstrated in our laboratory.

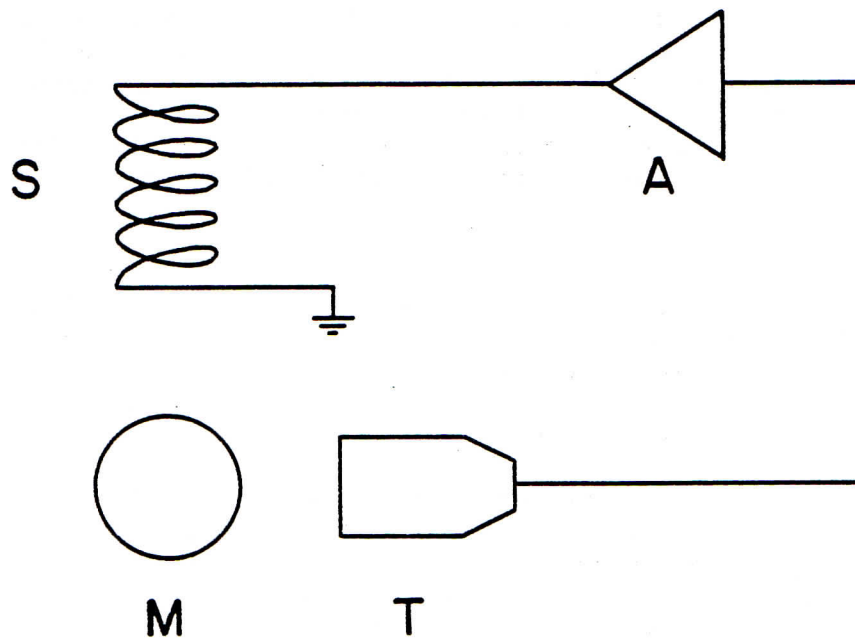
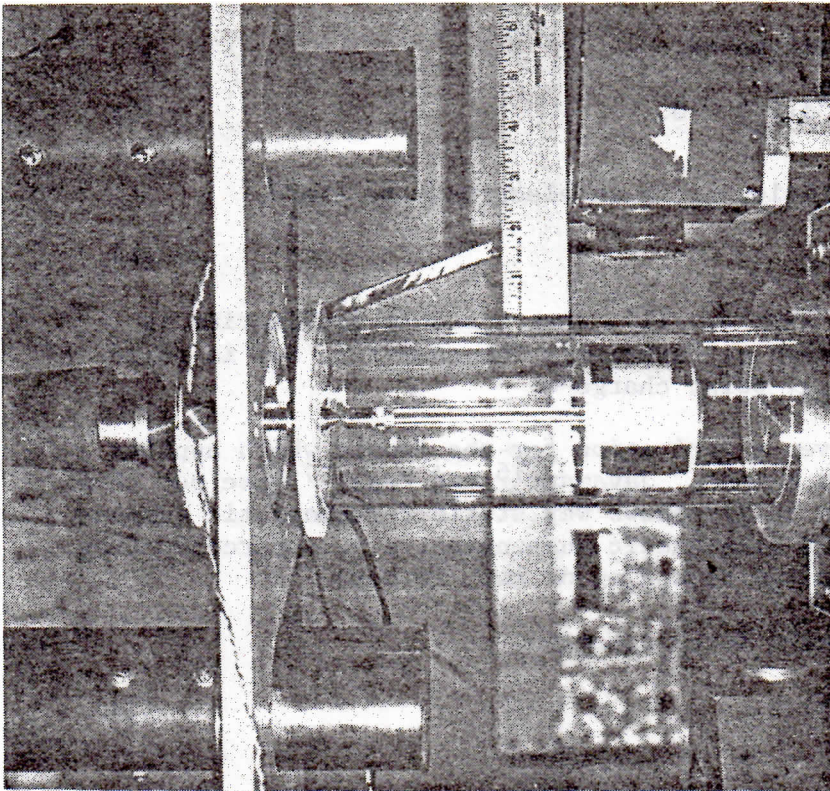


Figure 1

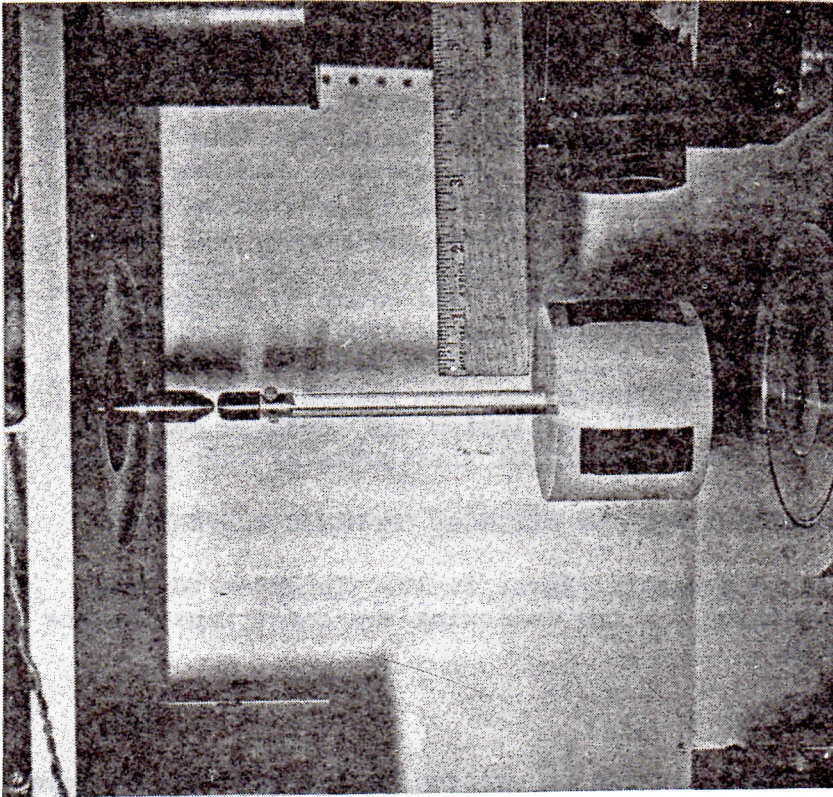
A general magnetic suspension system in which a ferromagnetic sphere is suspended. T is a position transducer, A is the feedback electronic circuitry which controls the current in the solenoid S.





A

An optically sensed magnetic suspension of a 250 gm rotor in a quartz chamber. Tube on the top left houses the optics of the light source, tube on the top right houses the detector and its optics. The permanent magnet assembly in the top middle provides the main lifting force on the rotor.



B

A close up of the suspended rotor. The tip of the rotor is made of Superinvar and is the only ferromagnetic part of the rotor.

Figure 2



While extensive analysis of electric and magnetic levitations is available<sup>12</sup>, we were unable to find a single simplified reference in a reasonably standard format and of the type that would be appropriate for needs such as ours. In particular, we wanted to eliminate details of the support coil current in the differential equations thus eliminating mixed variables with implicit connections. And, we avoid extensive servo analysis. We feel that magnetic suspension is an example of practical electricity and magnetism, mechanics, and linear control theory in a combination that most physics and engineering teachers and students may find interesting. For example, a magnetically suspended object undergoing small motions can be modelled as a damped harmonic oscillator. The role of magnetic forces and electronic circuitry (treated as functional blocks) in such motion will be discussed. In addition, mechanical equilibrium and feedback corrective forces provide examples of other mechanical attributes of this system.

## 2. THE EQUILIBRIUM CONDITION FOR THE VERTICAL MAGNETIC SUSPENSION

Consider a particle in a static force field. In order to have stable equilibrium at position  $\vec{r}_0$ , the following two conditions for the force  $\vec{F}$  must be satisfied,

$$\vec{F}(\vec{r}_0) = 0, \text{ and } \vec{\nabla} \cdot \vec{F}(\vec{r}_0) < 0. \quad (1)$$

If  $\vec{F}$  is an irrotational field, it can be written as the gradient of some potential  $\phi$ ,

$$\vec{F}(\vec{r}) = -\vec{\nabla}\phi(\vec{r}), \quad (2)$$

and Eq. (1) can be rewritten as,

$$\vec{\nabla}\phi(\vec{r}_0) = 0, \text{ and } \nabla^2\phi(\vec{r}_0) > 0. \quad (3)$$

For a charged particle,  $q$ , in an electrostatic field,

$$\vec{\nabla} \cdot \vec{F}(\vec{r}) = q\vec{\nabla} \cdot \vec{E} = 0, \quad (4)$$

which shows, following from Eq. (1), that a stable equilibrium is impossible. This is known as Earnshaw's Theorem<sup>13,14</sup>, and is valid only for a monopole, e.g. point charge  $q$ .

Baumbek<sup>15</sup>, has given a derivation for the dipole which applies to a suspended magnetic moment. Jayawant<sup>16</sup> and Papas<sup>17</sup> have put this in a concise form which easily shows why static ferromagnetic suspension is not possible but diamagnetic suspension, e.g. superconducting, is possible. We follow the reasoning of ref. 16 here.

We apply the magnetic force equation,

$$\vec{F}_m = \vec{\nabla} \left( \frac{1}{2} \int \vec{M} \cdot \vec{B} dv \right) \quad (5a)$$

to an extended dipole, in this case an induced one. The permanent dipole becomes a trivial subclass of this where

$$\vec{F}_m = \vec{\nabla}(\vec{M} \cdot \vec{B}). \quad (5b)$$

Then the induced magnetization is related to the magnetic intensity  $\vec{H}$  by,

$$\vec{M} = \chi_m \vec{H}, \quad (6a)$$

where  $\chi_m$  is the magnetic susceptibility of the body,  $\vec{H} = \vec{B}/\mu$  and  $\mu$  is the permeability. The induced dipole moment of the body is given by

$$\vec{M} = \int \vec{M} dV, \quad (6b)$$

over  $V$ , the volume of the body. Assuming the body is small enough for  $\vec{H}$  to remain constant throughout its volume

$$\vec{M} = \int \chi_m \vec{H} dV = \chi_m H V. \quad (6c)$$

From eq. (5),

$$\vec{F}_m = \vec{\nabla} \left( \frac{1}{2} \int \vec{M} \cdot \vec{B} dV \right) = \frac{1}{2} \mu \chi_m \vec{\nabla} H^2. \quad (6d)$$

Since the divergence of  $\vec{\nabla} H^2$  is nowhere negative the stable equilibrium condition, equa. (1) can only be met if  $\chi_m$  is negative, i.e. for a diamagnetic material. ( $\mu$  is always non-negative).

With the addition of the gravitational force, Eq. (5) becomes,

$$\ddot{m}\vec{r} = \left( \frac{1}{2} \int \vec{M} \cdot \vec{B} dV \right) + m\vec{g}, \quad (7)$$

which may create the equilibrium condition  $\ddot{m}\vec{r} = 0$ , but since  $\vec{\nabla} \cdot m\vec{g} = 0$ , the stability condition of Eq. (6) remains unchanged.

In the instances of interest here, both the applied field  $\vec{B}$ , and the dipole moment  $\vec{M}$ , have azimuthal symmetry. For the vertical magnetic suspension described by Fig. 3, it can be demonstrated in the laboratory (and has been discussed by Fremery<sup>18</sup>) that the unstable equilibrium is confined to the vertical,  $z$ , direction, and that there is a stable equilibrium with respect to the radial direction. This stable equilibrium in the radial direction is analogous to the radial equilibrium of a simple pendulum<sup>18</sup>, with the equilibrium position lying along the  $z$  axis. Therefore, for the remainder of this discussion only the equilibrium in the  $z$  direction will be discussed, and Eq. (7) reduces to

$$m\ddot{z} = \frac{\partial}{\partial z} \left( \frac{1}{2} \int \vec{M} \cdot \vec{B} dV \right) + mg, \quad (8)$$

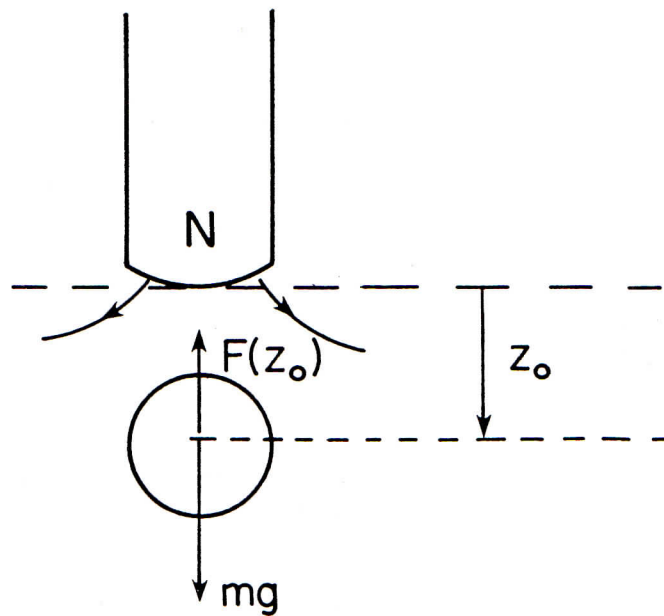


Figure 3

A ferromagnetic ball experiences a magnetic force  $F(z_0)$  when its center is located at a vertical distance  $z_0$  from the tip of the pole piece of a permanent or uncontrolled electro magnet. At  $z_0$  the upward magnetic force is equal to the downward gravitational force.



where  $z$  downward is positive. Stability in the  $z$  direction can only be achieved by introducing feedback control, for ferromagnetic suspension.

A practical suspension can be made in which the dipole moment of a suspended object is either permanent or induced. The ideal moments will be centered at the center of mass. These two idealized conditions of suspension will be analyzed separately.

$$1) \quad \vec{M} = \text{constant}$$

From Eq. (5b) it is apparant that the dipole moment,  $\vec{M}$ , will align itself parallel to the applied field,  $\vec{B}$ , in order to achieve the lowest potential energy. In many practical instances  $\vec{B}$  can be broken into two components: i) The permanent, static field,  $\vec{B}_p$ , due to any permanent magnet present plus that due to any direct current in the coil, and ii) the control field,  $\vec{B}_c(t)$ . That is,  $\vec{B} = \vec{B}_p + \vec{B}_c(t)$ , and for all the following analysis we require that  $B_p \gg B_c$ , where  $B$  is the norm of the vector. Therefore,  $\vec{M}$  aligns itself with  $\vec{B}_p$  and is considered to be time independent. (Fluctuations in its direction give pendulum motion.<sup>18</sup>) Both of these conditions have been verified by experiment. For a mass,  $m$ , supported along the vertical axis in a vacuum, the permanent moment equivalent of Eq. (8) becomes

$$m\ddot{z} = M \frac{\partial B_p}{\partial z} + \vec{M} \cdot \frac{\partial \vec{B}_c}{\partial z} + mg. \quad (9)$$

The conditions for control can be obtained from Eq. (9) by considering the various terms in connection with the equilibrium condition and departures from it. Let  $\langle \rangle$  represent time averages, then,

$$m\langle \ddot{z} \rangle = M \frac{\partial \langle B_p \rangle}{\partial z} + \vec{M} \cdot \frac{\partial \langle \vec{B}_c \rangle}{\partial z} + \langle mg \rangle, \quad (10)$$

where the first and third terms on the right are obviously constant in ordinary circumstances since they do not contain time varying parameters. However, a feedback system always operates with some control signal, there is no absolute (i.e. static) equilibrium but only a time averaged equilibrium. When critically damped, the suspended object undergoes fluctuations of amplitude no larger than some tolerated steady state error,  $z_{ss}$ , but its equilibrium position remains unchained over time. Correspondingly,  $B_c$  is a fluctuating quantity which responds to the mass's deviation from the average equilibrium position,  $z_0$ . Hence,  $\langle \ddot{z} \rangle = 0$ , and  $\langle \vec{B}_c \rangle = 0$ . This leads to the following result:

$$mg = -M \frac{\partial B_p}{\partial z}, \quad (11)$$

which is a restatement of the equilibrium condition. Hence, for a given  $m$  and  $M$ , a magnetic suspension in equilibrium means that



$$\frac{\partial \vec{B}}{\partial z} \cdot \vec{p} < 0, \quad (12)$$

and has a fixed magnitude. Equation (12) suggests that the density of the magnetic field lines must increase with decreasing  $z$ . An illustration of this is shown in Fig. 4. The core can have either polarity. Notice that the equilibrium condition is uniquely determined independently of the control field, and hence, the equilibrium position,  $z_0$ , is independent of the feedback control system. (Obviously, though, the position sensor must be located near  $z_0$ .) Therefore,  $\vec{B}_c$  only affects the dynamic operation of the suspension system in order to insure that the equilibrium condition is stable (this is discussed further in section 4 in the context of behavior of the suspension as a spring.)

$$b) \quad \vec{M} = \xi \vec{B}$$

For this case, Eq. (8) becomes,

$$m\ddot{z} = \frac{1}{2} \xi \frac{\partial}{\partial z} (\vec{B} \cdot \vec{B}) + mg. \quad (13)$$

And as before,  $\vec{B} = \vec{B}_p + \vec{B}_c$ , so that now

$$m\ddot{z} = \frac{\xi}{2} \frac{\partial B_p^2}{\partial z} + \frac{\xi}{2} \frac{\partial B_c^2}{\partial z} + \xi \frac{\partial}{\partial z} (\vec{B}_p \cdot \vec{B}_c) + mg. \quad (14)$$

Taking time averages,

$$m\langle \ddot{z} \rangle = \frac{\xi}{2} \frac{\partial B_p^2}{\partial z} + \frac{\xi}{2} \frac{\partial \langle B_c^2 \rangle}{\partial z} + \xi \frac{\partial}{\partial z} \langle \vec{B}_p \cdot \vec{B}_c \rangle + mg. \quad (15)$$

For stable suspension the positional variation of mass  $m$  must be small. This implies that  $B_c$  should be small and hence,  $\langle B_c^2 \rangle \approx 0$ . As before  $\langle \dot{z} \rangle = 0$  and  $\langle \vec{B}_c \rangle = 0$ . Therefore, the condition for equilibrium becomes

$$-\frac{\xi}{2} \frac{\partial B_p^2}{\partial z} = mg, \quad (16a)$$

so once again  $B_p$  is uniquely determined. Equation (16a) can be written as,

$$-\xi B_p \frac{\partial B_p}{\partial z} = mg, \quad (16b)$$

so again  $\frac{\partial B_p}{\partial z} < 0$ . This can also be written as,

$$-M \frac{\partial B_p}{\partial z} = mg, \quad (16c)$$

the same as Eq. (11).

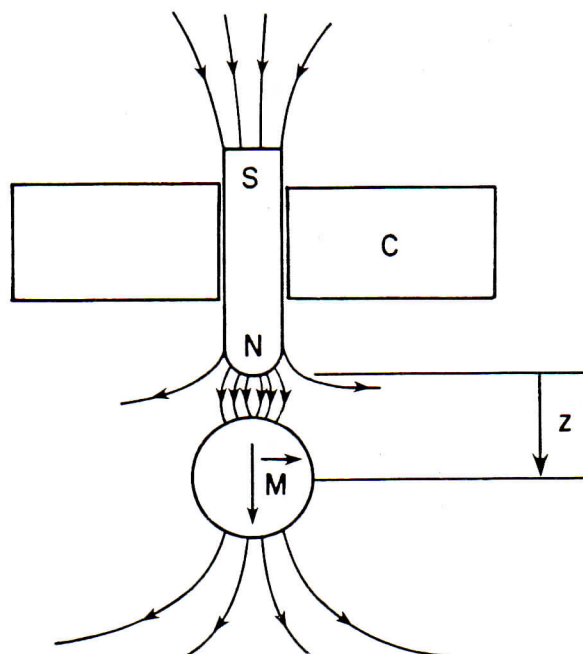


Figure 4

Illustration of flux causing suspension. If the polarity of the magnet is reversed all field lines reverse direction, as does  $M$ .

### 3. PRACTICAL CONSIDERATIONS ABOUT THE EQUILIBRIUM CONDITION

It is important to get a quantitative sense of the dependence of the vertical equilibrium position on the support field and coil current. For the case described in Consideration a) of the previous section, the result is totally dependent on the field gradient, which is nearly proportional to the current,  $I$ , unless saturation of the magnetic material in the pole piece occurs. For the case described in Part b), the suspension force is proportional to  $I^2$ , as can be seen from Eq. (16). For an object of high permeability and spherical geometry, such as a steel sphere of radius  $a$  (permeability  $> 100$ ),  $\xi$  is approximately  $a^3$ .<sup>19</sup> For a steel ball at equilibrium, Eq. (16b) becomes,

$$-a^3 B_p \frac{\partial B}{\partial z} = mg = \frac{4}{3} \pi a^3 \rho g, \quad (17)$$

where  $\rho$  is the density of the ball ( $\sim 7.8 \text{ gm/cm}^3$  for iron). A numerical expression for Eq. (17) is,

$$-B_p \frac{\partial B}{\partial z} = \frac{4}{3} \pi \rho g \approx 3.2 \times 10^4 \text{ gauss}^2/\text{cm} = 3.2 \times 10^{-2} \text{ T}^2/\text{m}. \quad (18)$$

This expression has been verified in one of our suspension systems. A 1.27 cm diam. steel ball was magnetically suspended with the aid of a permanent magnet. A cold rolled steel cup was made to match together the bottom of the permanent magnet and the top of the coil core so that most of the field lines pass through the core. Prior to suspension, a gauss-meter was employed to measure the magnetic field versus distance from the tip of the core, in steps of 0.1 cm over a 3 cm span. The quantities  $B_{av} \times \Delta B/\Delta z$  were then plotted versus  $z$  and are shown for one pole shape in Fig. 5. As shown, Eq. (18) was satisfied at  $z_0 = 1.1 \text{ cm}$ . Obviously, for a stronger magnet the line would move up and this would occur at a larger  $z_0$ . With the ball magnetically suspended, it was found that the center of the ball was located at  $1.3 \pm 0.1 \text{ cm}$  from the core tip when the direct current through the coil was essentially zero. The ability of Eq. (18) to fall within 20% of this position is considered verification, in view of the fact that the effective  $\vec{M}$  might not be at the center of the sphere, and given the approximations in measuring  $B_{av}$  and  $\Delta B/\Delta z$ . Note that such an observation is independent of the gain of the feedback system, as long as there are no oscillations. The correcting field can come from above or below the suspended object. In other words, the location of the coil is flexible and this flexibility comes in handy for some experiments.

### 4. THE MAGNETIC SUSPENSION AND DAMPED HARMONIC MOTION

Consider the force  $F(z)$  acting on an object of mass  $m$  by a given permanent or electro-magnet as a function of the axial distance  $z$  away from one end of the magnet as shown in Fig. 3. Suppose at  $z_0$ , we have

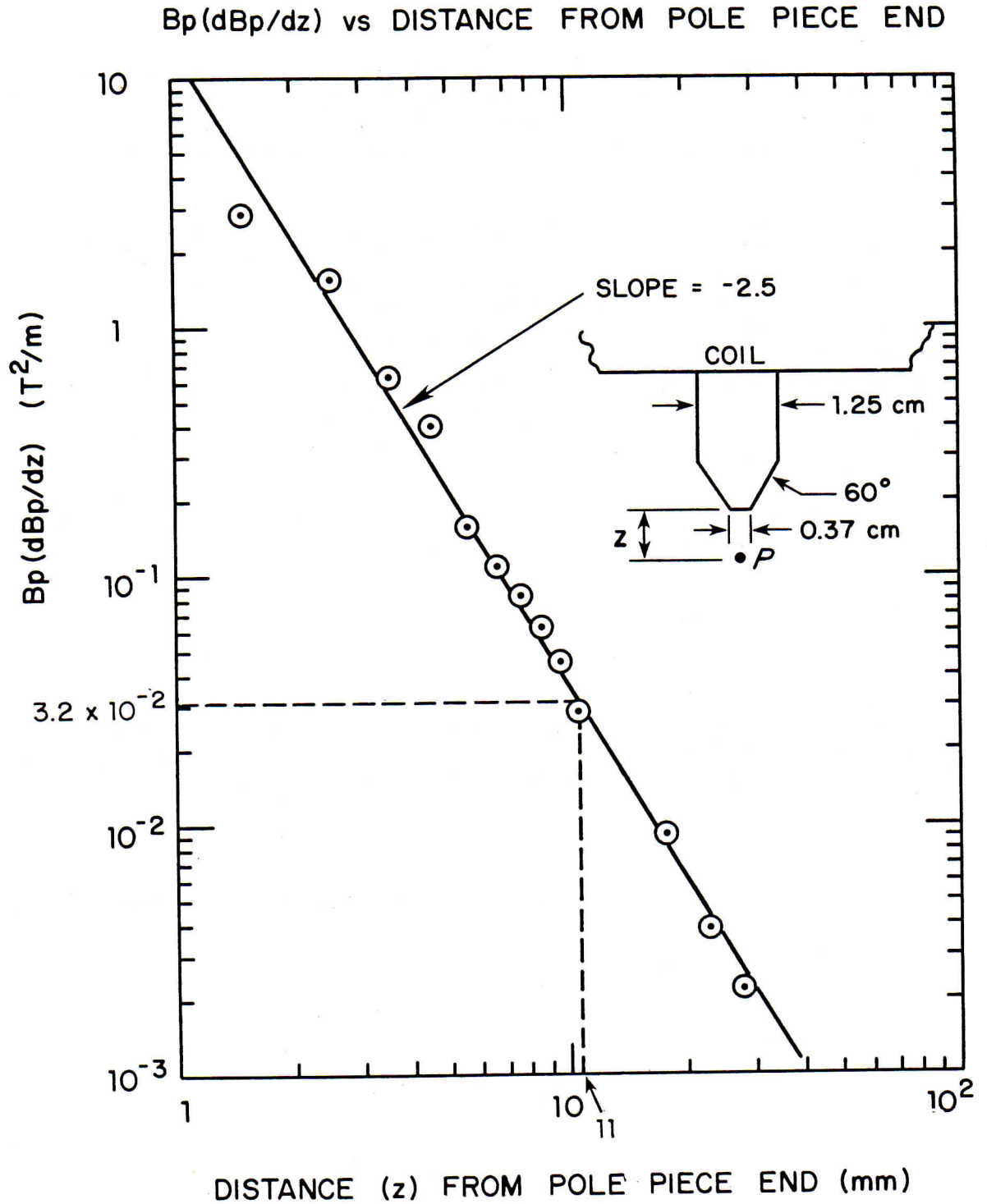


Figure 5

Plot of  $B_{av} \frac{\Delta B}{\Delta z}$  which is proportional to magnetic support force  $F(z)$  on suspended soft iron sphere. The equilibrium position  $z_0$  calculated from Eq. (18) is 11 mm.



$$mg = F(z_0). \quad (19)$$

Now, as discussed earlier,  $z_0$  is not a point of stable equilibrium because a positive (negative) deviation from  $z_0$  would cause the object to continue moving away from (nearer) the magnet. The potential  $U(z)$  of the combined magnetic-gravitational force is shown in curve a of Fig. 6.

This curve describes a "negative" spring which is another way to exhibit the impossibility of static equilibrium between the magnetic force and the gravitational force. According to our previous discussion, a positive spring is actually needed to restore the body to its original position. Such a positive spring is created by feedback such that the resultant potential will still have positive slopes after being combined with that of the negative spring. This is shown in curves b and c of Fig. 6. The steepness of the resultant potential reflects the sensitivity of the transducer and the "tightness" of the feedback loop.

The goal here is to model an ideally suspended body as a critically damped linear oscillator (with small error departures from equilibrium) whose motion is described by the equation

$$m\ddot{z} + b\dot{z} + kz = 0, \quad (20)$$

where  $m$  is the mass,  $b$  is the damping coefficient which we shall explain,  $k$  is the spring constant, and  $z$  is now redefined for simplicity as a displacement from  $z_0$ , where  $z_0$  is an absolute equilibrium position with respect to the tip of the pole piece; i.e.  $z = z_{\text{abs.}} - z_0$ , and  $z = 0$  at  $z_{\text{abs.}} = z_0$ . Equation (8) can be rewritten as,

$$\vec{F} = \frac{1}{2} \vec{\nabla}(\vec{M} \cdot \vec{B}) + mg. \quad (21)$$

Then the magnitude of the total force (without considering damping) is,

$$F = \frac{1}{2} |\vec{\nabla}(\vec{M} \cdot \vec{B})| + mg = -kz. \quad (22)$$

At equilibrium,

$$dF = -k dz = \frac{\partial}{\partial z} \frac{1}{2} |\vec{\nabla}(\vec{M} \cdot \vec{B})|_{z=0} dz, \quad (23)$$

and,

$$k = - \frac{\partial}{\partial z} \frac{1}{2} |\vec{\nabla}(\vec{M} \cdot \vec{B}_p)|_{z=0} - \frac{\partial}{\partial z} \frac{1}{2} |\vec{\nabla}(\vec{M} \cdot \vec{B}_c)|_{z=0}. \quad (24)$$

(Note that  $dz = dz_{\text{abs.}}$ ) The first term on the right of Eq. (24) is known to be negative as shown by Braunbek and Papas and corresponds to curve a of Fig. 6, as mentioned above. By definition,  $\vec{B}_c$  is only dependent on the changes in the current (voltage) of the electromagnet.



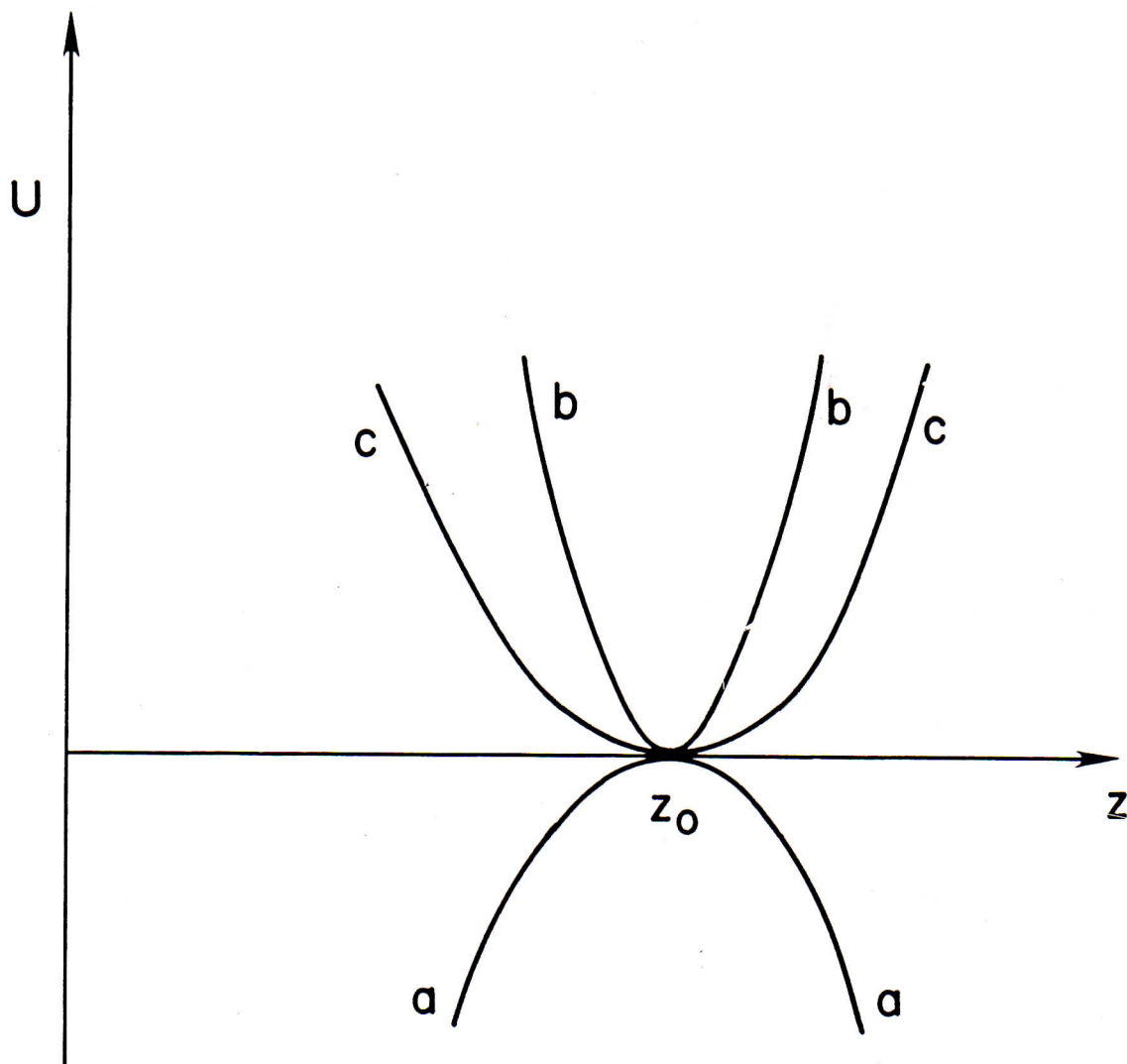


Figure 6

The combined magnetic-gravity potential functions encountered by a ferromagnetic object without feedback (curve a) and with feedback (curve c). Curve b describes a gravity-free magnetic potential function due to feedback over the linear region.  $z_0$  is the point where the magnetic force on the body exactly balances the force due to gravity.

Therefore, the second term of Eq. (24) can be written as,

$$- \frac{\partial}{\partial V} \frac{1}{2} |\vec{\nabla}(\vec{M} \cdot \vec{B}_c)|_{z=0} \frac{\partial V}{\partial z}, \quad (25)$$

where  $V$  is the voltage applied across the electromagnet, and  $\partial V / \partial z$  is known as the position-to-voltage transfer coefficient of the position transducer. Equation (25) must be the larger of the two terms of Eq. (24), and it must be positive in order to create a stable equilibrium in the vertical direction as described by curves b and c of Fig. 6. (Actually the terms of Eq. (24) can only be considered constants over a small range of  $z$  about  $z = 0$ .) Since the magnetic force will increase with increasing the voltage across the electromagnet,  $\partial |\vec{\nabla}(\vec{M} \cdot \vec{B}_c)| / \partial V|_{z=0} > 0$ , and therefore to have stable equilibrium,  $\partial V / \partial v < 0$ , i.e. negative feedback, so that voltage becomes more negative as position increases.

The damping may come from viscous coupling of  $m$  to the surrounding fluid (air or a liquid) or it can be created electronically. In one of our biophysics experiments<sup>11</sup>, a permanent magnet is suspended in an aqueous virus solution whose viscosity is to be measured by determining the phase lag of the magnetically driven, oscillating buoy. On the other hand, if the object is suspended in an evacuated environment to achieve high  $Q$  rotations, damping must be added electronically. As often encountered in mechanics or circuit theory, this damping (derivative) action brings an additional phase lag which is usually frequency dependent. An expression for the electronic time constant for critical damping is shown in the appendix.

It is sufficient for our purpose to point out that the control magnetic field,  $\vec{B}_c$  is actually a superposition of time varying fields of different frequencies, amplitudes and phases which together control the ultimate stability of the servo-system. If some components of these fields were absent or become phase-shifted (such as could happen by placing conducting material between the coil and the suspended object), the suspension could become sluggish or even impossible. Detailed study of this aspect is done in control theory.<sup>20,21</sup>

## 5. AMPLITUDE-MODULATION PRINCIPLE APPLIED TO MAGNETIC SUSPENSION

As pointed out in the introduction, there are quite a few detection schemes all of which have their vulnerabilities and limitations. In the case of the conventional optical detection (Fig. 7a) in which direct current is employed for the illumination of an incandescent light bulb or a LED (light emitting diode), it is obvious that any change of ambient light might be mistaken by the photo-detector as a change of control signal and would cause subsequent motion of the body. The low frequency drift ( $1/f$  noise) inherent in DC systems and the noise of electronic circuitry, especially at the front (detector) stage over a long period of time is also a disadvantage.

One method we have used to alleviate these problems in the optically sensed magnetic suspension is that of amplitude modulation.<sup>22</sup> As shown

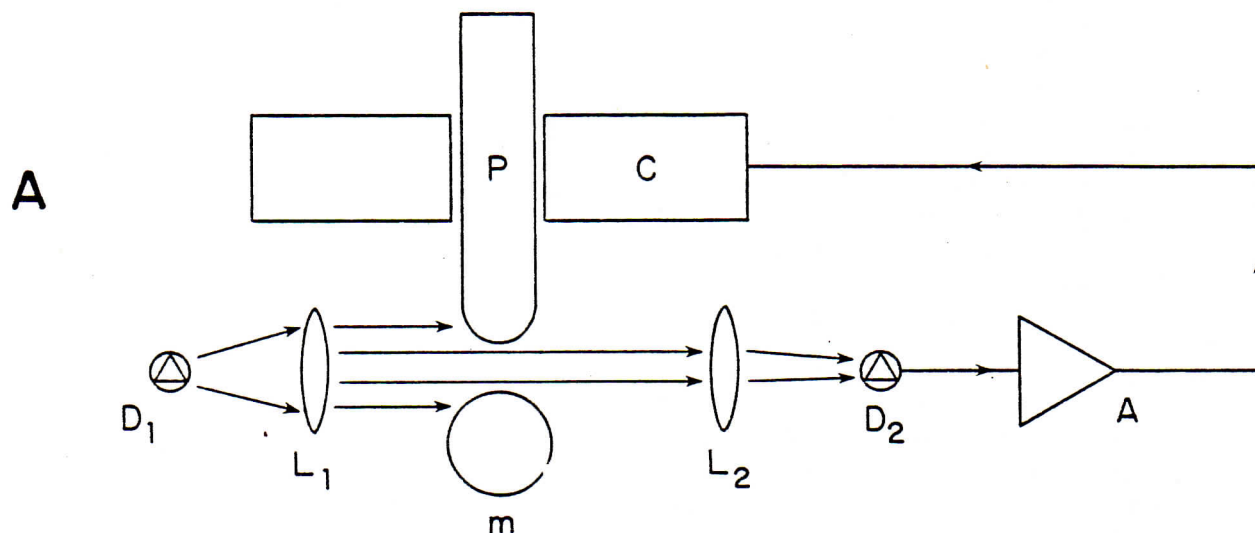


Figure 7A - Single diode optical sensing of magnetic suspension system. Light from light emitting diode  $D_1$  is focussed by lens  $L_1$  between pole piece  $P$  and suspended mass  $m$ , then focussed by lens  $L_2$  onto photodiode  $D_2$ . Height variations of  $m$  change current into amplifier-differentiator  $A$  which varies current to coil  $c$ .

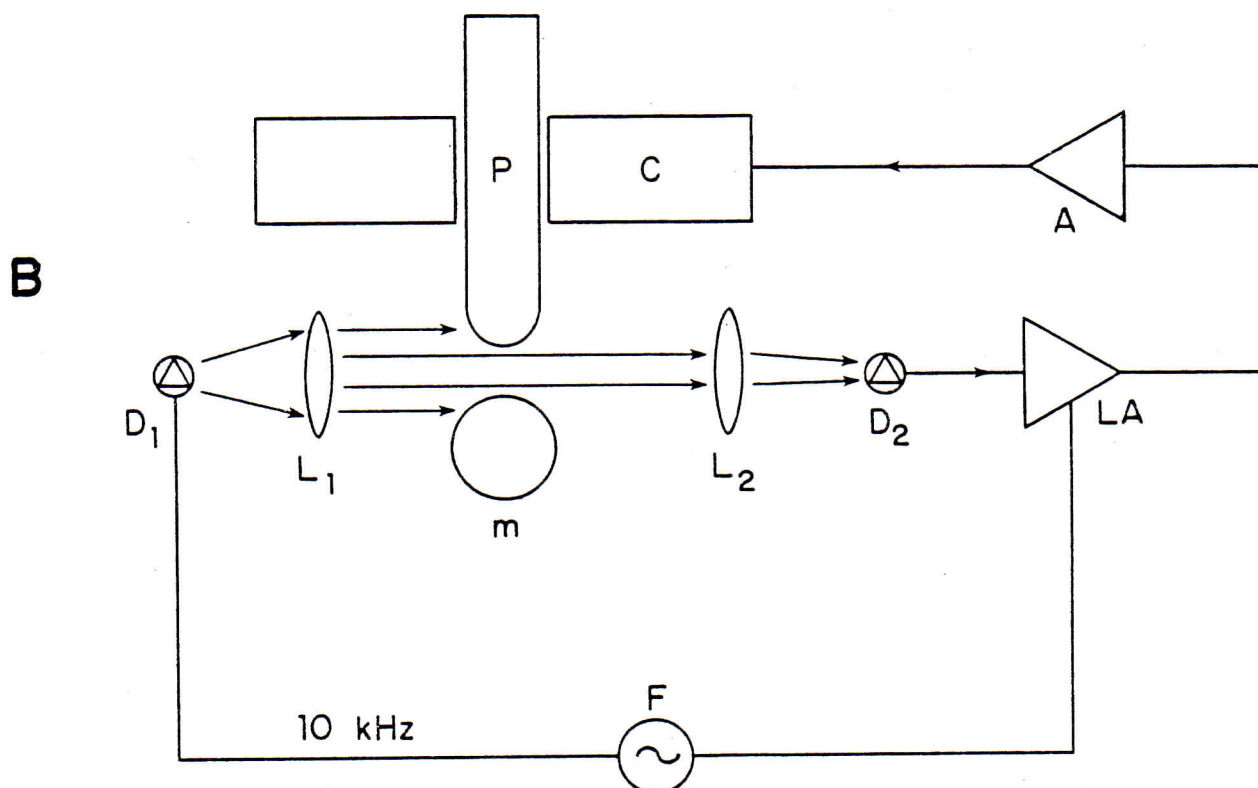


Figure 7B - Single diode optical sensing of magnetic suspensions with amplitude modulation. Frequency generator  $F$  pulses the light emitting diode  $D_1$ . Information about height variation is now contained in variations of amplitude component at this carrier frequency; such information is extracted by the lock-in amplifier  $LA$  whose reference frequency also comes from  $F$ .



in Fig. 7b, a carrier frequency (conveniently of 10 kHz) is chosen to pulse the light source. The error motion of the suspended object therefore amplitude-modulates this carrier frequency and is then "seen" by the photodetector as distinct from nonsynchronous ambient light. One can employ conventional AM circuitry or a lock-in amplifier to demodulate the desired signals. The latter case in which the reference frequency to the lock-in amplifier comes from the same frequency generator is known as the synchronous detection method. Other frequencies are averaged out, or "rejected" by the lock-in amplifier. With this improvement, the suspension system can be left in the open; room light fluctuations (especially the 120 Hz of fluorescent lights) do not bother the system as long as the photodetector is not saturated. Low frequency drift is reduced significantly since the noise spectrum is now moved from DC to the 10 kHz region. It must be pointed out that the pulsing frequency should be chosen so that it is above the mechanical frequency spectrum and yet within the frequency range of the signal processing circuitry. Details and quantitative advantages have been shown in a separate paper.<sup>23</sup>

## CONCLUSIONS

The vertical ferromagnetic suspension system can be analyzed as a damped harmonic oscillator. The system parameters, including the properties of the feedback loop, are describable in terms of the coefficients in the differential equation. It is possible and convenient to have the equation of motion exhibit separately the conditions of equilibrium and of control.

## ACKNOWLEDGEMENT

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## APPENDIX

It may be desirable to have a magnetic force on the suspended object equal to zero at some  $z_1$ , and equal to  $2mg$  at some  $z_2$ , where  $z_1 < z_0 < z_2$ . Then by building an appropriate solenoid<sup>1</sup> so that the magnetic control force is nearly linear over this distance,  $k(z_2 - z_1) = 2mg$  and  $m = k(z_2 - z_1)/2g$ . This imposes a constraint on  $k$  for a given mass and range of linearity. Without significant viscous forces,  $b$  is entirely electronic in nature, and since the position transducer converts position to voltage, we can describe the servo loop as a block diagram in the time domain as shown in Fig. 8. In the block diagram  $G_1$  is called the

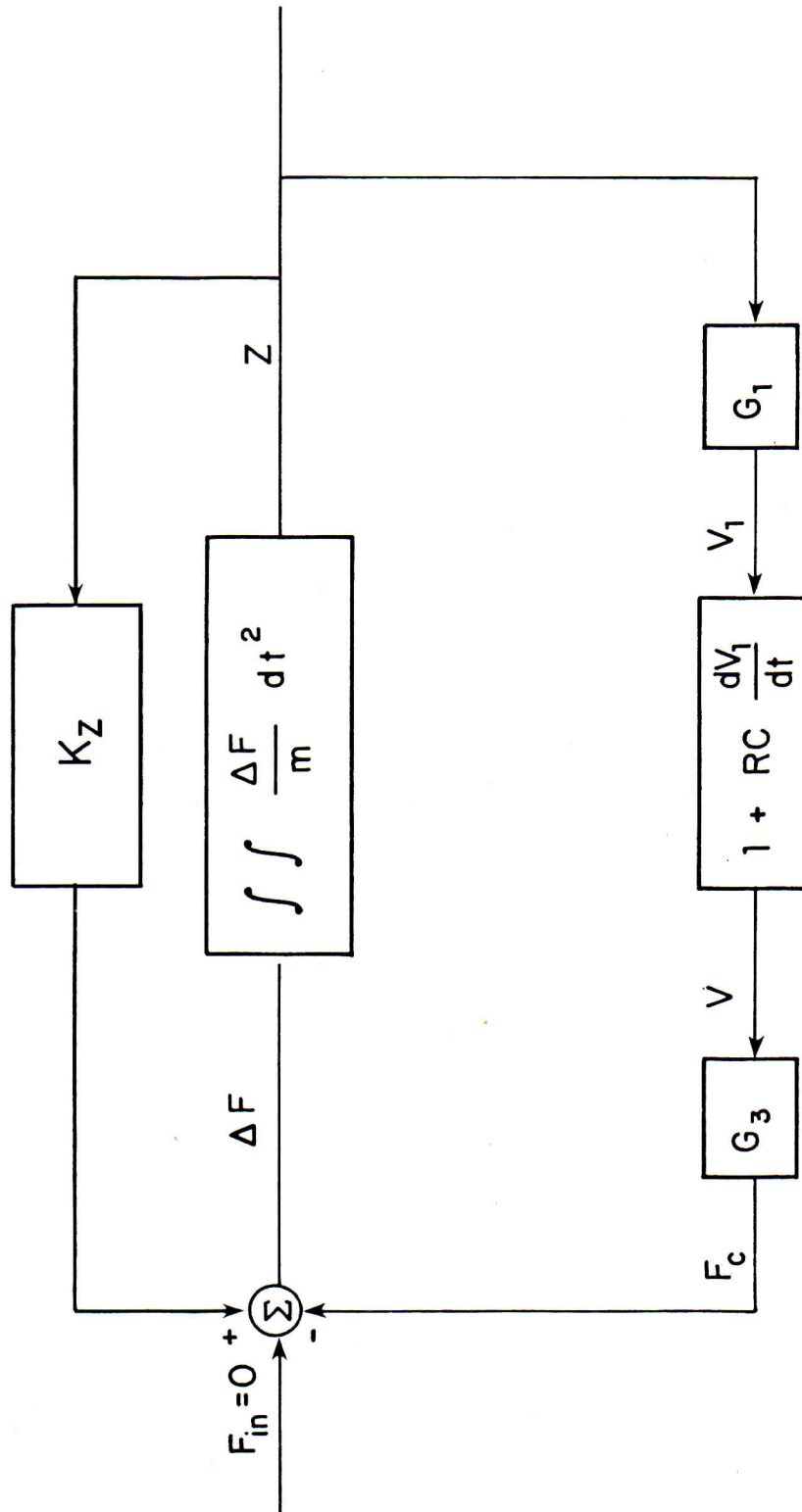


Figure 8

Time domain block diagram of the feedback loop of a simple suspension system.



position-to-voltage transfer function,  $G_3$  the voltage-to-force transfer function, and  $k_z$  is the position-to-force transfer function due to Braunbek's Theorem. Note that the input force is zero (i.e. at equilibrium) and that the control force is completely contained in the feedback path, hence the control force acts as the so-called "error signal",  $\Delta F$ , which is being driven to zero. From Fig. 8, Eq. (18) becomes

$$m \frac{\ddot{V}_1}{G_1} + G_3 G_1 RC \frac{\dot{V}_1}{G_1} + (G_3 G_1 - K_z) \frac{V_1}{G_1} = 0, \quad (23)$$

where  $G_1 z = V_1$ ,  $k = (G_3 G_1 - K_z)$ , and now,  $m = (G_3 G_1 - K_z)(z_2 - z_1)/2g$ . For the system to be critically damped,

$$RC = 2 \left[ \frac{(G_3 G_1 - K_z)^2 (z_2 - z_1)}{2g G_3^2 G_1^2} \right]^{1/2} \quad (24)$$

A workable value for  $RC$ , however, can be obtained by realizing that  $G_3 G_1 > K_z$ , and therefore,

$$RC \approx 2[(z_2 - z_1)/2g]^{1/2}. \quad (25)$$

In suspensions of the type we use, only a small operating range is possible, that is, a reasonable value for  $z_2 - z_1$  is 5 mm, for which  $R \approx 40$  msec. Referring again to Fig. 5, we see that a range much greater than 5 mm would be impractical for such a coil and pole piece. It is possible, however, to use huge coils, yokes, and pole pieces to attain considerably different regimes of operation, if desired.

It is important to emphasize that in this context the control force,  $F_c$ , is only due to a displacement from equilibrium, and is not the total "lifting force". Similarly,  $z$  is a displacement from the equilibrium position,  $z_0$ , and is not an absolute position.

<sup>†</sup>Present address: Hanson Laboratories, Stanford University, Palo Alto, California 94305 (USA)

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